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Markov Decision Process in no-data Problem based on Probabilitic Differental Equation in Fuzzy Events

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Abstract: *Tanaka et al formulated the fuzzy-Bayes decision making rule by integral transformation based on the expected utility maximization theory as an extension to Wald's subjective modification fuzzy event. Hori et al formulated the fuzzy · Bayes decision making rule which extended Wald's decision function to fuzzy OR combination and fuzzy AND combination with many subjective distribution. This decision-making law is based on the state of nature Is a decision-making rule after mapping and conversion to fuzzy events, and it is an OR type 2 fuzzy by mapping fuzzy functions such as subjective distribution and utility functions to fuzzy events. Furthermore, Hori introduced the Markovian time concept to the state of nature, and derived the Markov process and Markov decision process in fuzzy events. This is a natural extension to the stochastic process theory of Wald's decision function, and the fuzzy event of appearance of the natural state becomes a Markov process having a fuzzy transition matrix, and as a result of the Monte Carlo simulation, the annihilation, reversal, resurrection Repeat the cycle. Finally, Hori et al proposed an illusion state identification method as an example of adaptation of these fuzzy / Bayes decision making rules. In addition, Hori et al. Firstly used the max product method by mapping / transformation of membership functions of fuzzy events in a fuzzy event in which the subjective distribution and utility function in the no data problem transit like ergodic Markov We formulated these Markov decision processes. Note that this series of flows is a natural extension to the stochastic process of Wald's decision function. Next, we consider subjective distribution and utility function as fuzzy functions, subjectivity maps / converts natural state by subjective distribution, utility assumes that natural state is mapped / converted by utility function, The subjectivity and the utility also showed that it follows Markov process. Finally, the subjectivity and utility in fuzzy events were propagated by Markov processes in which each element of the transition matrix follows the Markov process, and proposed the Markov decision process by the Max product method*

Keywords: *Mapping technique, stochastic differential equations, Ito integral, type 2 fuzzy, type 3 fuzzy*

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I. Introduction

In Fuzzy-Bayes decision making rule, we consider two distributions of subjectivity distribution which expresses human subjectivity and membership function which converts and maps state of nature into fuzzy event by membership function. The membership function is a kind of filter function that is set by each decision maker and transforms and maps the natural state into fuzzy events with membership functions. As this filter function is set by the subjectivity of each decision maker, consequently giving a certain degree of freedom to fuzzy events. With Zadeh, a fuzzy set was proposed, and Tanaka et al extended it to a decision-making problem. This extension is called fuzzy Bayes decision making rule, but it was shown by Hori that it is included in the subjective modification of the Wald function. This indicates that fuzzy · Bayes decision making rule is included in subjective Bayes theory. However, Hori has mentioned that Type 2 fuzzy is unique in fuzzy-Bayes decision making law. In addition, Markov decision process in the fuzzy event after the mapped and transformed the state of nature by the decision-making membership function, subjectivity after mapping the state of nature by subjective distribution and utility function Markov decision process in utility was derived. In this paper, the subjective view is a subjective distribution of the state of nature, after conversion and mapping, and utility regards the state of nature as a utility function after conversion and mapping, and map technique in the expansion principle of The mapping We formulates the stochastic differential equations based on. Furthermore, as a mapping of the mapping, fuzzy theory formalizes the stochastic differential equations based on the subjective distribution and the utility function in the fuzzy event as type 3 fuzzy. Here, it is considered that solutions of these stochastic differential equations follow Ito's integral. As a future subject, I would like to study the effectiveness and validity of fuzzy theory by actually guiding the difference in optimal behavior when not introducing fuzzy logic theory from Ito integral and Max product law. In particular, by this formulation, when the membership function is an identity function, the probability differential equations are equal when introducing fuzzy logic and not introducing it. This proves that fuzzy logic is one type of subjective Bayes theory when the decision maker is a dangerous neutral person

II. Fuzzy-Bayes decision rule in type-2 fuzzy events

We assume that the subjective distribution Π(S) and utility function U(S|D) have been identified by the decision maker by lottery, for state of nature S, where D denotes actions. Using a fuzzy polynomial regression model to envelop the observed information, we denote the upper,

central, and lower fuzzy events as *μF1(S), μF2(S), and μF3(S)*, respectively. We further assume that the fuzzy risk for the state of nature is *μR1(S), μR2(S), or μR3(S)* when the decision maker is respectively risk-tolerant, risk-neutral, or risk-averse.

Applying Zadeh's extension principle of mapping, we derive three types of subjective possibility distribution and fuzzy utility functions as

$$\mu_{F_1}(\Pi^{-1}(S)),\mu_{F_2}(\Pi^{-1}(S)),\mu_{F_3}(\Pi^{-1}(S)) \qquad (1)$$

$$\mu_{F_1}(U^{-1}(S/D)),\mu_{F_2}(U^{-1}(S/D)),\mu_{F_3}(U^{-1}(S/D)) \qquad (2)$$

The upper, central, and lower levels respectively represent risk-tolerant, risk-neutral, and risk-averse fuzzy events, and we accordingly propose deriving the fuzzy information quantities with respect to risk, defining the possibility measures in the integrated type-2 fuzzy events and taking the maximum as the optimal action as follows.

$$\begin{aligned} W_{R_1} &\triangleq \max_s \mu_{R_1}(S)\log\mu_{R_1}(S) \\ W_{R_2} &\triangleq \max_s \mu_{R_2}(S)\log\mu_{R_2}(S) \\ W_{R_3} &\triangleq \max_s \mu_{R_3}(S)\log\mu_{R_3}(S) \end{aligned} \qquad (3)$$

$$\begin{aligned} &\Pi_D \triangleq \\ &W_{R1}\cdot\max_s \mu_{F_1}(\Pi^{-1}(S))\cdot\mu_{F_1}(U^{-1}(S/D)) \\ &+W_{R_2}\cdot\int\mu_{F_2}(\Pi^{-1}(S))\cdot\mu_{F_2}(U^{-1}(S/D))ds \\ &+W_{R_3}\cdot\max_s \min\left(\mu_{F_3}(\Pi^{-1}(S)),\mu_{F_3}(U^{-1}(S/D))\right) \\ &D^*=\max_D \Pi_D \end{aligned} \qquad (4)$$

As an alternative method for deriving the integrated possibility measure for fuzzy events identified as dome-shaped distribution from the observed information, we perform an α-level cut of the fuzzy events. Since the upper and lower fuzzy events are respectively risk-tolerant and risk-averse, we can then formulate the decision rule as shown in Eq. (5). As a two-objective programming problem, this yields innumerable Pareto solutions, and we therefore incorporate a fuzzy goal such as Eq. (6), derive the integrated possibility measures, and take the result with the maximum possibility measure as the optimal action.

$$\begin{cases} \max_{\alpha} \max_{s} \mu_{F_1\alpha}(\Pi^{-1}(S)) \cdot \mu_{F_1\alpha}(U^{-1}(S/D)) \\ \min_{\alpha} \max_{s} \mu_{F_3\alpha}(\Pi^{-1}(S)) \cdot \mu_{F_3\alpha}(U^{-1}(S/D)) \end{cases} \tag{5}$$

$$\begin{cases} \max_{s} \mu_{F_1\alpha=0}(\Pi^{-1}(S)) \cdot \mu_{F_1\alpha=1}(U^{-1}(S/D)) \\ \max_{s} \mu_{F_1\alpha=1}(\Pi^{-1}(S)) \cdot \mu_{F_1\alpha=0}(U^{-1}(S/D)) \end{cases}$$

$$\begin{cases} \max_{s} \mu_{F_3\alpha=1}(\Pi^{-1}(S)) \cdot \mu_{F_3\alpha=0}(U^{-1}(S/D)) \\ \max_{s} \mu_{F_3\alpha=0}(\Pi^{-1}(S)) \cdot \mu_{F_3\alpha=1}(U^{-1}(S/D)) \end{cases} \tag{6}$$

3. Markov process in no-data problem

We take $\{yt\}$ as the Markov process and $\{Ft\}$ as the Markov decision process in fuzzy events, and show the Markov-like membership function$\{\mu t_{F_t}(yt)\}$ in the ordinary Markov process

$$D_{y_t} = L(t, yt) \tag{7}$$

The Markov process D_{F_t} in fuzzy events may be considered a Markov process after mapping and transformation of the transition matrix (7) of the ordinary Markov process by the Markov-like membership function $\{\mu t_{F_t}(yt)\}$. By the principle of extension of Zadeh mapping, we then have

$$D_{F_t} \triangleq \int \mu t_{F_t}(yt)/D_{y_t} = \int \mu t_{F_t}(yt)/L(t,yt)$$
$$= \max_{\{xt=L(t,yt)\}} \mu t_{F_t}(yt) = L^{-1}(t, \mu t_{F_t}(xt))$$

where

$$L^{-1}L(t,yt) = I \tag{8}$$

We now perform an α-level cut of the elements of each fuzzy number in the Markov process in fuzzy events, writing α-or for performance of the or linking.Accordingly, the α-or union of the discrete series $\{xt\}$ may be considered a transition matrix of the Markov process in fuzzy events, and thus may be defined as

$$\bigcup_{\alpha=0}^{1} \underset{\{xt=L\alpha(t,yt)\}}{\alpha\text{-}or} = \underset{\{xt=L(t,yt)\}}{or}$$
$$\triangleq \max_{\{xt=L(t,yt)\}} \mu t_{F_t}(yt) = L^{-1}(t, \mu t_{F_t}(xt)) \tag{9}$$

Note that Eqs. (8) and (9) are equivalent, which shows not only that fuzzy mathematics can be applied in the Markov process but also that fuzzy statistics is included in subjective Bayesian statistics. It shows, moreover, that fuzzy comprises a type of stochastic differential equation.

The initial and convergence conditions are shown here as Theorems 1 and 2, with Pos and Nes as indicators of possibility and necessity.

Theorem 1 *Conditions* $(1) Ft0 = Zt0$
$(2) Pos(Ft \geqq Zt|x0) \leqq Pos(DFx0 \geqq DZx0)$
$(3) Nes(Ft \geqq Zt|x0) \geqq Nes(DFx0 \geqq DZx0)$
where $DFx0 = DZx0$

Theorem 2 *Conditions* $(1) Ft0 = Zt0$
$(2) Pos(Ft \geqq Zt) \leqq Pos(DF0 \geqq DZ0)$
$(3) Nes(Ft \geqq Zt) \geqq Nes(DF0 \geqq DZ0)$
where $DF0 = DZ0$

As an example of an application of this Markov process with the transition matrix of this Markov process as an identity matrix in a case in which the Markov-like membership function is a Gaussian process, the Markov process in fuzzy events is then a Gaussian process. The proof of this was derived by Uemura by incorporating the concept of time, based on its linear state and its normal distribution.

4. Markov decision in no-data problem

Into the Markov process in fuzzy events described in we incorporate the utility function $\{Ut_{At}(yt)\}$ for transformation in a Markov-like manner. Here, the membership function $\{FUt_{At}(yt)\}$ of the fuzzy utility in fuzzy events is represented by Eq. (10) by the mapping extension principle, and in the same manner as discussed in the fuzzy utility $\{D_{FUt}\}$ is derived by Eq. (14) with $\{At\}$ as the decision process.

$$
\begin{aligned}
FUt_{At}(yt) &= \int \mu t_{Ft}(yt) \,/\, Ut_{At}(yt) \\
&\triangleq \max_{\{xt=Ut_{At}(yt)\}} \mu t_{Ft}(yt) \\
&= \mu t_{Ft}(U^{-1}t_{At}(xt))
\end{aligned}
$$

Here,

$$U^{-1}Ut_{At}(yt)=1 \tag{10}$$

$$
\begin{aligned}
D_{FUt} &= \int FUt_{At}(yt) \,/\, D_{y_t} \\
&= \int FUt_{At}(yt) \,/\, L(t,yt) \\
&\triangleq \max_{\{zt=L(t,yt)\}} FUt_{At}(yt) \\
&= L^{-1}(t, FUt_{At}(zt))
\end{aligned}
$$

Here,

$$L^{-1}L(t,yt)=I \tag{11}$$

As the possibility measure for this decision process we use the max-product operation and define it by the following equation.

(Definition of possibility measure of decision process)

$$\Pi_{At} \triangleq \max D_{Ft} \times D_{FUt} \tag{12}$$

5. Markov decision in type-2 fuzzy event

Hori and Matsumoto proved that if the state of nature can be assumed to follow the Markov process of a transition matrix $L(t, \chi t)$, then the fuzzy events also follow a Markov process with a fuzzy transition matrix. The fuzzy Markov processes of the upper, central, and lower levels are then

$$
\begin{aligned}
&L^{-1}(t,\mu_{F_{1t}}(\chi t)), L^{-1}(t,\mu_{F_{2t}}(\chi t)), \\
&L^{-1}(t,\mu_{F_{3t}}(\chi t))
\end{aligned} \tag{13}
$$

With the fuzzy event membership function $\mu 4t(\chi t)$ as in Eq. (14) after weighting for fuzzy information quantity, the Markov process in the type-2 fuzzy events can be expressed as in Eq. (18).

$$\begin{aligned}\mu_{4t}(\chi t) \triangleq\, & W_{R_1}\cdot\mu_{F_{1t}}(\chi t)\\ & +W_{R_2}\cdot\mu_{F_{2t}}(\chi t)\\ & +W_{R_3}\cdot\mu_{F_{3t}}(\chi t)\end{aligned} \tag{14}$$

$$D_t = L^{-1}(t,\mu_{4t}(\chi t)) \tag{15}$$

We next incorporate the decision process At with one decision for each process and denote as $UtAt(\chi t)$ the utility function weighted for fuzzy information quantity. The fuzzy utility function is then derived as Eq. (16) by the mapping extension principle, and its Markov decision process is

expressed as in Eq. (17).

$$F_{UtAt}(\chi t) = \mu_{4t}(U^{-1}tAt(\chi t)) \tag{16}$$

$$D_{FUt} = L^{-1}(t, F_{UtAt}(\chi t)) \tag{17}$$

We calculate the possibility measure using the max-product operation, and take the result with the maximum possibility measure as the optimal action.

$$\begin{aligned}\Pi_{Dt} &= \max_{\chi t} D_t \times D_{FUt}\\ D^*_t &\triangleq \max_{Dt} \Pi_{Dt}\end{aligned} \tag{18}$$

6. Extension to multidimensional process

In two-dimensional natural states (S_1, S_2) with prior possibility distribution $\Pi\ (S_1, S_2)$, and membership function for human subjectivity $\mu_{Fj}\ (S_1, S_2)$, by assuming the natural states are mutually independent, we obtain Eqs. (19) and (20). We note that the membership function maps and transforms the natural states to fuzzy events Fj.

$$\mu_{Fj}(S_1,S_2) = \mu_{Fj}(S_1) \wedge \mu_{Fj}(S_2) \tag{19}$$

$$\Pi(S_1,S_2) = \Pi(S_1) \times \Pi(S_2) \tag{20}$$

The fuzzy event possibility measure $\Pi(Fj)$ can then be derived by the max-product operation.

$$\Pi(Fj) = \max_{(S_1,S_2)} \mu_{Fj}(S_1,S_2) \times \Pi(S_1,S_2) \tag{21}$$

We incorporate the two-dimensional utility function $U_{Di}(S1,S2)$ with actions Di. It has been shown that, by applying the extension principle of Zadeh's mapping, the fuzzy utility function can be derived by the following equations, and, with the utility function, Zadeh's extension principle is thus already valid and effective.

$$\begin{aligned}U(Fj,Di) &= \int \mu_{Fj}(S_1,S_2) \,/\, U_{Di}(S_1,S_2)\\ &= \sup_{(S1',S2')=U_{Di}(S1,S2)} \mu_{Fj}(S_1,S_2)\\ &= \mu_{Fj}(U^{-1}{}_{Di}(S_1',S_2'))\end{aligned} \tag{22}$$

From Eqs. (24) and (25), the fuzzy expected possibility measure $\Pi_E\,(Di)$ is then

$$\Pi_E(Di)=\sum_j \max_{(S_1,S_2)} \Pi(Fj)\times U(Fj,Di) \tag{23}$$

The optimum action is the action yielding the maximum fuzzy expected possibility measure in Eq. (23).

$$D*\triangleq \max_i \Pi_E(Di) \tag{24}$$

Function U^{-1} is the inverse of the utility function U and function L^{-1} is the inverse of the L transition rate matrix, and in cases yielding multiple inverse functions and transition rate matrices, fuzzy union can be applied following operation on each inverse function by the extension principle.

In this investigation, we assume that transitions in the two Markov processes represented by Eq. (25) are mutually independent. The problem under consideration is the high-risk high-return decision problem and is thus risk-tolerant, and can be expressed as the two-dimensional Markov process in Eq. (23).

$$D_{yt}=L_1(t,yt),Dzt=L_2(t,Zt) \tag{25}$$

$$D\{yt,zt\}=L_1(t,yt)\vee L_2(t,Zt) \tag{26}$$

If we express the membership function series transforming and mapping these two Markov processes to their respective fuzzy events by Eq. (27), the Markov processes in these fuzzy events can then be derived by Eqs. (28) and (29).

$$\{\mu_{tFt}(\chi_{1t}),\mu_{tFt}(\chi_{2t})\} \tag{27}$$

$$L_1(t,yt)=L^{-1}(t,\mu_{tFt}(\chi_{1t})) \tag{28}$$

$$L_2(t,zt)=L^{-1}(t,\mu_{tFt}(\chi_{2t})) \tag{29}$$

Next, incorporating the two decision processes $\{A_1t, A_2t\}$ and taking their utility functions as $U_{1tA_1t}(\chi_1t)$ and $U_{2tA_2t}(\chi_2t)$, respectively, we find the fuzzy utility functions by Eqs. (30) and (31) and obtain the Markov utility functions for fuzzy events by Eqs. (32) and (33).

$$F_{U_{1t}A_{1t}}(yt)=\mu_{tFt}(U^{-1}_{1tA_{1t}}(\chi_{1t})) \tag{30}$$

$$F_{U_{2t}A_{2t}}(zt)=\mu_{tFt}(U^{-1}_{2tA_{2t}}(\chi_{2t})) \tag{31}$$

$$D_{FU_{1t}}=L^{-1}(t,F_{U_{1t}A_{1t}}(yt)) \tag{32}$$

$$D_{FU_{2t}}=L^{-1}(t,F_{U_{2t}A_{2t}}(zt)) \tag{33}$$

The two Markov processes are independently ergodic, and, in the case of high-risk high-return decision problems, the Markov decision processes for two-dimensional fuzzy events can therefore be obtained by

$$D_{F(U_1,U_2)t} = D_{FU_{1t}} \vee D_{FU_{2t}} \tag{34}$$

$$\Pi_h = \max_{(S_1,S_2)} D\{yt,Zt\} \times D_{F(U_1,U_2)t} \tag{35}$$

$$D^*_h \triangleq \max_h \Pi_h \tag{36}$$

7. Another Markov decision in fuzzy events

It has been shown that if the state of nature Dt follows the Markov process of transition matrix L according to Eq. (37), then the fuzzy events Ft which are its manifestations follow the Markov process expressed in Eq. (38), where $\mu_{F_t}(\chi t)$ is the fuzzy event membership function and L^{-1} is the inverse matrix of the transition matrix.

$$D_t = L(t,\chi t) \tag{37}$$

$$F_t = L^{-1}(t,\mu_{F_t}(\chi t)) \tag{38}$$

In the no-data problem, if it is assumed that the subjective distribution Π_t and utility function $U_{t,D}$ are identified sequentially by decision-maker lottery, then after mapping and transformation, the subjective possibility distribution and the fuzzy utility function in fuzzy events can be separately derived using the extension principle as

$$\begin{aligned} \Pi_t &\triangleq \sup_{\{yt=\Pi(\chi t)\}} L^{-1}(t,\mu_{F_t}(\chi t)) \\ &= L^{-1}(t,\mu_{F_t}(\Pi^{-1}(yt))) \end{aligned} \tag{39}$$

$$\begin{aligned} U_{t,D} &\triangleq \sup_{\{yt=U(\chi t\mid D)\}} L^{-1}(t,\mu_{F_t}(\chi t)) \\ &= L^{-1}(t,\mu_{F_t}(U^{-1}(yt\mid D))) \end{aligned} \tag{40}$$

where Π^{-1} is the inverse function of the subjective distribution and U^{-1} is the inverse function of the utility function. Note also that it has been shown using subjective Bayes theory and utility function theory that the extension principle is effective.

It has been demonstrated that fuzzy inference is appropriate for high-risk high-return decision problems when used in concert with the max-product method. The possibility measure $\Pi\Pi_t$ found by the max-product method is shown in Eq. (41). We next propose the Markov decision process with weighting by the fuzzy events by using this possibility measure and taking the maximum as the optimum action as in Eq. (42).

$$\Pi\Pi_t \triangleq \max \Pi_t \times U_{t,D}$$
$$= L^{-1}(t, \max\{\mu_{F_t}(\Pi^{-1}(y_t)) \times \mu_{F_t}(U^{-1}(y_t \mid D))\}) \quad (41)$$

$$D* = \max_{D} \Pi\Pi_t \times F_t \quad (42)$$

The subjective distribution and the utility function in the no-data problem are determined by decision-maker lottery based on certainty equivalence, and can therefore be regarded as piecewise linear fuzzy functions with fuzzy OR-connectives. Accordingly, subjectivity may be regarded as mapping and transformation of the state of nature by the subjective distribution and utility may be regarded as mapping and transformation of the state of nature by the utility function. Applying the mapping extension principle, the subjectivity and the utility thus both follow Markov processes, as

$$\Pi_t = L^{-1}(t, \Pi_t(\chi t)) \quad (43)$$

$$U_{t,D} = L^{-1}(t, U_{t,D}(\chi t)) \quad (44)$$

We next formulate the Markov decision process in fuzzy events. Subjectivity $F\Pi_t$ in fuzzy events can be regarded as subjectivity mapped and transformed by the membership function of fuzzy events, which can be derived as in Eq. (45) by the mapping extension principle, and utility $FU_{t,D}$ is similarly given by Eq. (46).

$$F\Pi_t \triangleq \sup_{\{y_t = L^{-1}(t, \Pi t(\chi t))\}} L^{-1}(t, \mu_{F_t}(\chi t))$$
$$= L^{-1}(t, L^{-1}(t, \mu_{F_t}(\Pi_t^{-1}(y_t)))) \quad (45)$$

$$FU_{t,D} \triangleq \sup_{\{y_t = L^{-1}(t, U_{t,D}(\chi t))\}} L^{-1}(t, \mu_{F_t}(\chi t))$$
$$= L^{-1}(t, L^{-1}(t, \mu_{F_t}(U_{t,D}^{-1}(y_t)))) \quad (46)$$

We apply the max-product method and formulate the Markov decision process in fuzzy events as

$$D* \triangleq \max_{D} \max F\Pi_t \times FU_{t,D} \quad (47)$$

8. Stocastic differential equation in no-data problem

In equation (48), the subjective St is considered to be a map of the natural state St mapped and transformed by the subjective distribution $\Pi t(St)$. In addition, the subjective distribution is identified piecewise linearly by lottery, and it is a fuzzy OR bond. Therefore, the extension principle of Zadeh can be applied, and the subjectivity can be formulated by stochastic differential equations as follows.

$$\begin{aligned}\frac{d\Pi}{dt} &= \sup_{\{Zt=\Pi t(St)\}} b(t,St)+\sigma(t,St)\cdot Wt \\ &= b(t,\Pi_t^{-1}(Zt))+\sigma(t,\Pi_t^{-1}(Zt))\cdot Wt\end{aligned} \tag{48}$$

Likewise, since the lower utility function $Ut(St/D)$ given the decision D is also identified by lottery, it can be regarded as a fuzzy OR combination, fuzzy mathematics is applied and the utility U t is It can be formulated by stochastic differential equations.

$$\begin{aligned}\frac{dU}{dt} &= \sup_{\{Zt=Ut(St/D)\}} b(t,St)+\sigma(t,St)\cdot Wt \\ &= b(t,U_t^{-1}(Zt/D))+\sigma(t,U_t^{-1}(Zt/D))\cdot Wt\end{aligned} \tag{49}$$

In the no data problem, both subjectivity and utility are fuzzy OR combinations, so the decision making rule becomes a high risk high return problem of subjectivity and utility, and it is defined

by the Max product method by the following equation. Here, it is assumed that equations (51) and (49) are solved by Ito integral.

$$\max_{D}\max_{St} \Pi t(St)\cdot Ut(St/D) \tag{50}$$

Equations (48) and (49) above can be considered Type 2 fuzzy. Furthermore, we introduce the concept of fuzzy events to this decision-making problem. In this case, taking subjectivity and utility in the fuzzy event after mapping and converting the natural state by the decision-making membership function $\mu t(St)$, the subjectivity in the fuzzy event can be expressed as fuzzy The utility in the event is given by equation (52)

It is formulated as stochastic differential equations. Here we note that these are type 3 fuzzy.

$$\begin{aligned}F\Pi t(Zt) &= \sup_{\{Zt=F\Pi t(St)\}} b(t,St)+\sigma(t,St)\cdot Wt \\ &= b(t,\mu t(\Pi_t^{-1}(Zt)))+\sigma(t,\mu t(\Pi_t^{-1}(Zt)))\cdot Wt\end{aligned} \tag{51}$$

$$\begin{aligned}FUt(Zt/D) &= \sup_{\{Zt=FUt(St)\}} b(t,St)+\sigma(t,St)\cdot Wt \\ &= b(t,\mu t(U_t^{-1}(Zt/D)))+\sigma(t,\mu t(U_t^{-1}(Zt/D)))\cdot Wt\end{aligned} \tag{52}$$

Next, solving equations (51) and (52) with Ito integral, it is a high risk high return problem, so according to the Max product method, these decision rules are defined by the following equations.

$$\max_{D}\max_{St} F\Pi t(St)\cdot FUt(St/D) \tag{53}$$

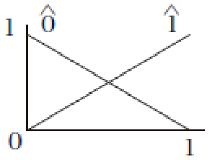
Here, when the membership function is an identity function, equations (48), (51), (49) and (52) have the same value,

The optimal behavior obtained by Eq. (53) is the same. This proves that fuzzy theory is included in the subjective Bayesian theory regardless of arbitrary operation when the decision maker is a dangerous neutral person. In the future, in the case where the membership function is convex upward (the decision maker is a dangerous person) and the case where the membership function is convex downward (decision maker is a risk avoider), actually two different probability derivatives We are going to solve the equation and consider each optimal behavior.

9. Adaption example 1 - next-stage state discrimination problem

We consider the transition from an independent state having two dimensions through the two-dimensional elimination (N1), inversion (N2), and restoration (N3) states and assume the following transition matrices in which $\hat{0}$ and $\hat{1}$ are the fuzzy numbers zero and one, respectively.

$$[\hat{0},\hat{0}][\hat{0},\hat{1}][\hat{1},\hat{1}]$$



$$\triangleq [L_h(S_1,S_2)] \quad (h=1,2,3) \qquad (54)$$

From Eqs. (19), (20), and (21) and by referring to, we find the fuzzy possibility measures of states at the next time step with the following equation, and identify the next-stage state as the state exhibiting the largest fuzzy possibility measure.

$$\Pi_{t+1}(F,N_h) = \max_{(S_1,S_2)} L_h(S_1,S_2) \times \mu_F(S_1,S_2) \times \Pi(S_1,S_2) \qquad (55)$$

$$N^* \triangleq \max_h \Pi_{t+1}(F,N_h) \qquad (56)$$

Further, with the two-dimensional states taken as mutually exclusive, Eq. (58) then takes the form of Eq. (57) and the optimum next-stage state identification takes the form of Eq. (61).

$$\Pi'_{t+1}(F,N_h) = \max_{S_1} L_h(S_1) \times \mu_F(S_1) \times \Pi(S_1) + \max_{S_2} L_h(S_2) \times \mu_F(S_2) \times \Pi(S_2) \qquad (57)$$

$$N^* \triangleq \max_h \Pi'_{t+1}(F,N_h) \qquad (58)$$

10. Adaption example 2 - integrated possibility measure

Here we assume that the decision maker has subjectively assigned membership functions *($\mu_{F1}(S)$, $\mu_{F2}(S)$, and $\mu_{F3}(S)$)* to fuzzy events (risk-averse F1, risk-neutral F2, and risk-tolerant F3), relative to the risk or gain. The fuzzy event information quantity may then be defined as

$$Wi(S) \triangleq \mu_{Fi}(S) \times \log \mu_{Fi}(S) \quad (i=1,2,3) \qquad (59)$$

We can then take Eq. (60) as the integrated possibility measure weighted by the possibility measure and the fuzzy information quantity in each fuzzy event, and identify the optimum action as the one with the largest integrated possibility measure.

$$\begin{aligned} \Pi(Dj) = &\max_S \{\mu_{F_1}(U^{-1}Dj(S)) \times W_1(S)\} \wedge \Pi(S) \\ &+ \int \{\mu_{F_2}(U^{-1}D_j(S)) \times W_2(S)\} \times \Pi(S)ds \\ &+ \max_S \{\mu_{F_3}(U^{-1}D_j(S)) \times W_3(S)\} \times \Pi(S) \\ D^* \triangleq &\max_j \Pi(Dj) \end{aligned} \qquad (60)$$

11. Conclusion

In this paper, the conventional stochastic differential equations are considered fuzzy by introducing the state of nature, the probability differential equation using the mapping technique is Type 2 fuzzy, and furthermore, the stochastic differential based on the fuzzy event using the mapping technique We considered that the equation is type 3 fuzzy. As a future task, we aim to build a Markov decision process in fuzzy events in the no data problem by actually solving equations (48), (49), (51) and (52) with Ito . Finally, we hope that our efforts can contribute to future artificial intelligence research by clarifying the relationship between type 2 fuzzy and type 3 fuzzy and stochastic differential equations.

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